

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Introduction to Quantum Mechanics

Subject Code : 4SC06QMC1

Branch: B.Sc. (Physics)

Semester : 6

Date : 23/04/2018

Time : 02:30 To 05:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1	Attempt the following questions:	(14)
	a) Which kind of energy is possessed by the free particle?	01
	b) Define a Wave packet.	01
	c) What is Heisenberg's uncertainty principle? Write its equation.	01
	d) What do you mean by the normalization of a wave function ψ with finite norm?	01
	e) What are the Admissibility Conditions of a wave function?	01
	f) Define the Stationary States.	01
	g) What is a momentum Eigen function? How can it be normalized?	01
	h) Write the formula for the reflection probability \mathcal{R} at potential barrier and walls.	01
	i) Define the adjoint of an operator.	01
	j) What is self adjoint operator?	01
	k) What are the properties of a self adjoint operator?	01
	l) What is the Dirac Delta function?	01
	m) Fill in the blanks from Ehrenfest theorem: $m \frac{dr}{dt} = \underline{\hspace{2cm}}$ and $\frac{d\langle P_r \rangle}{dt} = \underline{\hspace{2cm}}$	01
	n) Write the formula for the energy eigen values of the simple harmonic oscillator.	01

Attempt any four questions from Question No. 2 to Question No. 8

Q-2	Attempt all questions	(14)
	A Obtain Schrödinger's equation for free particle in one dimension. From that, obtain it for three dimensions.	06
	B Obtain Schrödinger's equation for a particle in a force field.	04
	C Derive : Time dependent Schrödinger Equation.	04



Q-3	Attempt all questions	(14)
A	Discuss about the Physical interpretation for a wave function $\Psi(r, t)$ and probability interpretation for finding a particle in three dimensional space.	07
B	Write a short note on the Normalization of a wave function ψ .	04
C	Normalize the given wave functions.	03
	$\Psi = a \cdot \exp i(kx - \omega t)$ where $-2 \leq x \leq 2$	
	$\chi = \exp(-i\theta)$ where $0 < \theta < 2\pi$	
	$\Psi'(\phi) = A \sin(mn\phi)$ where $0 < \phi < 2\pi$	
Q-4	Attempt all questions	(14)
A	Explain giving example: Normalization of wave function with infinite norm by the method of Box Normalization.	07
B	Explain the conservation of probability for normalizable wave functions.	07
Q-5	Attempt all questions	(14)
A	Write a short note on Expectation Value derive necessary formulas for the expectation value of (i) Energy (ii) Potential Energy and (iii) Momentum.	07
B	Derive : Time Independent Schrödinger Equation.	07
Q-6	Attempt all questions	(14)
A	Give a brief explanation of Eigen Function and its Eigen Values. Show that the wave functions $\psi(x) = \sin 2x$ and $\phi(z) = e^{3x}$ are eigen functions of $\partial^2 / \partial x^2$.	05
B	Describe the Square-Well Potential with one dimensional Schrödinger equation for different regions.	03
C	Discuss the particle under bound state ($\vec{E} < 0$) in a square well and derive its solutions.	06
Q-7	Attempt all questions	(14)
A	Give a detailed account for Energy Eigen Function for Even & Odd Parity with graphical representation.	08
B	Write the statements of three fundamental postulates for the particle waves in quantum mechanics. Discuss any one of them.	06
Q-8	Attempt all questions	(14)
A	For a given wave function $\psi = (\pi a^3)^{-1/2} \cdot e^{-r/a}$ in spherical polar coordinate system (r, θ, ϕ), prove that $\langle r \rangle = \frac{3a}{2}$; where $a = \text{constant}$.	04
B	For a wave function $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$, prove that the expectation value of x for a particle in a box with volume L^3 is $\langle x \rangle = \frac{L}{2}$.	04
C	Prove that : $[x, P_x^n] = n i \hbar P_x^{n-1}$ $[L_x, L_y] = i\hbar L_z$	04
	$[y, P_y^n] = n i \hbar P_y^{n-1}$ $[L_y, L_z] = i\hbar L_x$	
	$[z, P_z^n] = n i \hbar P_z^{n-1}$ $[L_z, L_x] = i\hbar L_y$	
D	Prove that the commutator of the position and momentum do not vanishes for a particle; as $[x, P_x] = [y, P_y] = [z, P_z] = i \hbar$	02

